Kaon Electromagnetic Form Factor within the k_T Factorization Formalism and It's Light-Cone Wave Function

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Abstract

We present a systematical study on the kaon electromagnetic form factors $F_{K^{\pm},K^0,\bar{K}^0}(Q^2)$ within the k_T factorization formalism, where the transverse momentum effects, the contributions from the different helicity components and different twist structures of the kaon light-cone (LC) wave function are carefully analyzed for giving a well understanding of the hard contributions at the energy region where pQCD is applicable. The right power behavior of the hard contribution from the higher helicity components and from the higher twist structures can be obtained by keeping the k_T dependence in the hard amplitude. Our results show that the k_T dependence in LC wave function affects the hard and soft contributions substantially and the power-suppressed terms (twist-3 and higher helicity components) make an important contribution below $Q^2 \sim several\ GeV^2$ although they drop fast as Q^2 increasing. The parameters of the proposed model wave function can be fixed by the first two moments of its distribution amplitude and other conditions. By varying the first two moments $a_1^K(1GeV)$ and $a_2^K(1GeV)$ with the region of 0.05 ± 0.02 and 0.10 ± 0.05 respectively, we find that the uncertainty of the kaon electromagnetic form factor is rather small.

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I. INTRODUCTION

The electromagnetic form factors provide useful information concerning the internal structures of the mesons, and they also provide useful platforms to check the rightness of the perturbative QCD (pQCD) theory. Recently, the electromagnetic form factor $F_\pi(Q^2)$ has been restudied in Refs.[1, 2, 3]. It was shown that when all the power suppressed contributions, which include higher order in α_s, higher helicities and higher twists in the light-cone (LC) wave function, and etc., have been taken into account, then the hard contributions can fit the present experimental data well at the energy region where pQCD is applicable. By comparison the behavior of the kaon electromagnetic form factor is less certain both experimentally and theoretically. The kaon is composed by two quarks with different quark masses, therefore it becomes a little more complicated to obtain its LC wave functions and to compute its electromagnetic form factor. For example, the kaon electromagnetic form factor has been studied in Refs.[4, 5] in the light-cone quark model only within the soft region. Here we shall present a systematical study on the charged/neutral kaon electromagnetic form factors in the intermediate and large energy region within the k_T factorization formalism by properly taking the $SU_f(3)$ breaking effects into account.

The kaon electromagnetic form factor can be obtained through the definition

$$\langle K(p')|J_\mu|K(p)\rangle = (p+p')_\mu F_K(Q^2), \tag{1}$$

where K stands for $K^\pm$, K^0 and $\bar{K}^0$ respectively, the vector current $J_\mu = \sum_i e_i \bar{q}_i \gamma_\mu q_i$, with the quark flavor i and the relevant electric charge e_i. The momentum transfer $q^2 = -Q^2 = (p-p')^2$ is restricted in the space-like region. In the LC quantization and by using the Drell-Yan-West ($q^+ = 0$) frame [6], the kaon electromagnetic form factor can generally be expressed as

$$F_K(Q^2) = \hat{\Psi} \otimes \hat{\Psi} = \sum_{n,\lambda_i} \sum_j e_j \int [dx_i][d\mathbf{k}_{i\perp}]_n \Psi_n^*(x_i, \mathbf{k}_{i\perp}, \lambda_i) \Psi_n(x_i, \mathbf{k}_{i\perp} + \delta_i q_\perp, \lambda_i), \tag{2}$$

where the summation extends over all quark/gluon Fock states which have a non-vanishing overlap with the kaon, e_j is the electric charge of the struck quark, Ψ_n are the corresponding wave functions which describe both the low and the high momentum partons, $[dx_i][d\mathbf{k}_{i\perp}]_n$ is the relativistic measure within the n-particle sector and $\delta_i = (1-x_i)$ or $(-x_i)$ depending on whether i refers to the struck quark or a spectator, respectively.

Similar to the pionic case, it can be found that the nominal power law contribution to $F_K(Q^2)$ as $Q \to \infty$ is $F_K(Q^2) \sim 1/(Q^2)^{n-1}$ in the light-cone gauge ($A^+ = 0$) [7], under the condition that n quark or gluon constituents are forced to change direction. Thus only the $q\bar{q}$ component of $\Psi^{((1-x)Q)}(x, \mathbf{k}_\perp, \lambda)$ contributes at the leading $1/Q^2$. For the large Q^2 region, the hard contribution to the kaon electromagnetic form factor can be written as

$$
\begin{aligned}
&F_K(Q^2) = \\
&\sum_j e_j \int [dx][dy][d^2\mathbf{k}_\perp][d^2\mathbf{l}_\perp] \Psi^{*(1-x)Q}(x, \mathbf{k}_\perp, \lambda) T_H(x, y, \mathbf{q}_\perp, \mathbf{k}_\perp, \mathbf{l}_\perp, \lambda, \lambda') \Psi^{(1-y)Q}(y, \mathbf{l}_\perp, \lambda') + \cdots
\end{aligned}
\tag{3}
$$

where the ellipses represent the higher Fock states' contributions, $[dx] = dx_1 d_2 \delta(1 - x_1 - x_2)$ and $[d^2\mathbf{k}_\perp] = d^2\mathbf{k}_\perp/16\pi^3$. $\Psi^{((1-x)Q)}(x, \mathbf{k}_\perp, \lambda)$ is the valence Fock-state LC wave function with helicity λ and with a cut-off on $|\mathbf{k}_\perp|$ that is of order $(1-x)Q$. Such a cut-off on $|\mathbf{k}_\perp|$ is necessary to ensure that the wave function is only responsible for the lower momentum region. And the hard scattering amplitude T_H contains all two-particle irreducible amplitudes for $\gamma^* + q\bar{q} \to q\bar{q}$.

The LC wave function provides useful links between the hadronic phenomena in QCD at large distance (non-perturbative) and small distance (perturbative). A LC wave function is a localized stationary solution of the LC schrödinger equation $i\partial|\Psi(\tau)\rangle = H_{LC}|\Psi(\tau)\rangle$ [8, 9], which describes the evolution of a state $|\Psi(\tau)\rangle$ on the LC time $\tau \equiv x^+ = x^0 + x^3$ in the physical LC gauge $A^+ = 0$. For the valence quark state of the kaon, its LC wave functions can be defined in terms of the bilocal operator matrix element [10],

$$
\begin{aligned}
&\langle K(p)|\bar{q}_\beta(z) q_\alpha|0\rangle = \\
&\frac{i\sqrt{6}}{2}\left\{ \not{p}\gamma_5\, \Psi_K(x, \mathbf{k}_\perp) - \mu_K \gamma_5 \left[\Psi_p(x, \mathbf{k}_\perp) - i\sigma_{\mu\nu} \left(n^\mu \bar{n}^\nu \,\frac{\Psi'_\sigma(x, \mathbf{k}_\perp)}{6} - p^\mu \,\frac{\Psi_\sigma(x, \mathbf{k}_\perp)}{6} \,\frac{\partial}{\partial \mathbf{k}_{\perp\nu}} \right) \right] \right\}_{\alpha\beta}
\end{aligned}
\tag{4}
$$

where μ_K is the phenomenological parameter: $\mu_K = M_K^2/(m_s + m_u)$ for $K^\pm$ and $\mu_K = M_K^2/(m_s + m_d)$ for K^0 or $\bar{K}^0$ respectively, which is a scale characterized by the chiral perturbation theory. $\Psi_K(x, \mathbf{k}_\perp)$ is the leading twist (twist-2) wave function, $\Psi_p(x, \mathbf{k}_\perp)$ and $\Psi_\sigma(x, \mathbf{k}_\perp)$ are sub-leading twist (twist-3) wave functions that correspond to the pseudo-scalar structure and the pseudo-tensor structure respectively. The wave function $\Psi(x, \mathbf{k}_\perp)$ (Ψ stands for Ψ_K, Ψ_p and Ψ_σ respectively) satisfies the normalization condition

$$
\int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \Psi(x, \mathbf{k}_\perp) = \frac{f_K}{2\sqrt{6}}, \tag{5}
$$

where the decay constant $f_K = 160MeV$ [11]. The distribution amplitude (DA) $\phi(x)$ and the wave function $\Psi(x, \mathbf{k}_\perp)$ are related by

$$\phi(x) = \frac{2\sqrt{6}}{f_K} \int_{|\mathbf{k}_\perp|<\mu_f} \frac{d^2\mathbf{k}_\perp}{16\pi^3} \Psi(x, \mathbf{k}_\perp). \tag{6}$$

Non-leading perturbative contributions to the kaon electromagnetic form factor include the higher order in α_s, higher helicities and higher twists in the LC wave function, and etc. Similar to the pionic case [1, 2], it is substantial to take k_T dependence in the wave function into account and to keep the transverse momentum dependence fully in the hard scattering amplitude in the k_T factorization formalism within the LC framework. In present paper, we shall calculate all the helicity components' contributions to the kaon electromagnetic form factor within the LC pQCD framework, which is consistent with the using of LC wave function. Another important power correction is from the higher twist structures in the kaon DA. The end-point singularity becomes more serious for the higher twist structures, because the asymptotic behavior of the twist-3 DAs, especially $\phi_p^{as}(x) = 1$, so the calculations for these higher twist contributions have more uncertainty than that for the leading twist. It means that one should use the twist-3 wave function with a better behavior in the end-point region than that of the asymptotic one so as to give a more reliable estimation of the higher twist structures' contribution. The Brodsky-Huang-Lepage (BHL) prescription provides a useful way to construct a wave function with better end-point behavior [12], we shall adopt it to construct the kaon LC wave functions for the present purpose, and then we discuss its uncertainty for the kaon electromagnetic form factor. The $SU_f(3)$-breaking effects shall also be included for constructing the kaon LC wave function.

The reminder of the paper is organized as follows. Sec.II is devoted to present the main properties of the kaon electromagnetic form factor and the formulae for the twist-2 and twist-3 contributions to the kaon electromagnetic form factor within the k_T factorization approach. Numerical results for the kaon electromagnetic form factor are presented in Sec.III. The last section is reserved for conclusion and summary.

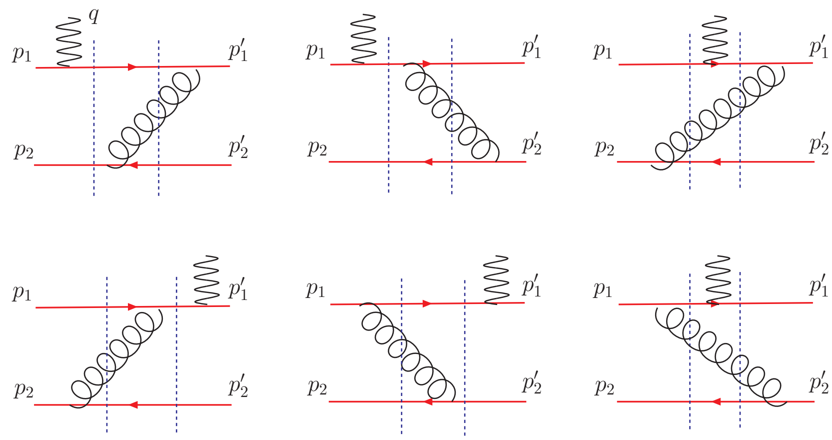

FIG. 1: Six leading order time-ordered Feynman diagrams for the hard scattering amplitude T_H, where $p_1 = (x_1, \mathbf{k}_\perp)$, $p_2 = (x_2, -\mathbf{k}_\perp)$, $p_1' = (y_1, y_1\mathbf{q}_\perp + \mathbf{l}_\perp)$, $p_2' = (y_2, y_2\mathbf{q}_\perp - \mathbf{l}_\perp)$.

II. KAON ELECTROMAGNETIC FORM FACTOR WITHIN THE k_T FACTORIZATION FORMALISM

Because, $K^+ = u\bar{s}$ and $K^- = s\bar{u}$, one can find that $F_{K^+} = -F_{K^-}$ according to Eq.(2). Similarly, since $K^0 = d\bar{s}$ and $\bar{K}^0 = s\bar{d}$, it can be found that $F_{K^0} = -F_{\bar{K}^0}$. So, we only need to calculate the K^+ and K^0 form factors, where $e_1 = 2/3$, $e_2 = 1/3$, $m_1 = m_u$ and $m_2 = m_s$ for F_{K^+} and $e_1 = -1/3$, $e_2 = 1/3$, $m_1 = m_d$ and $m_2 = m_s$ for F_{K^0} respectively. Here $m_u = m_d \neq m_s$ stand for the light constitute quark masses. Further more, in doing the calculation of the hard scattering amplitude with the k_T factorization formulism, we shall treat the current quark masses of u, d and s to be zero due to their smallness in comparison to the involved hard scale. Then the calculation procedure for the hard scattering amplitude is the same as that of the pionic case [1, 2].

A. Formulae for the twist-2 contribution to the kaon electromagnetic form factor

In the intermediate and large energy region, one can apply the pQCD approach and use the valence Fock state to estimate the kaon electromagnetic form factor since the applicability of pQCD in the intermediate and large energy region has been proved by involving the k_T dependence [13]. The lowest-order contribution for the hard scattering amplitude T_H comes from the one-gluon exchange Feynman diagrams as shown in Fig.(1). To simplicity

TABLE I: Full form of the LC wave function $\Psi(x, \mathbf{k}_\perp, \lambda) = \varphi(x, \mathbf{k}_\perp)\chi$. $\Psi(x, \mathbf{k}_\perp, \lambda)$ stands for $\Psi_K(x, \mathbf{k}_\perp, \lambda)$, $\Psi_p(x, \mathbf{k}_\perp, \lambda)$ and $\Psi_\sigma(x, \mathbf{k}_\perp, \lambda)$, respectively.

$\lambda_1\lambda_2$	$\uparrow\uparrow$	$\uparrow\downarrow$
$\Psi_{\lambda_1\lambda_2}(x, \mathbf{k}_\perp, \lambda)$	$-\frac{(a_1+a_2)(k_x-ik_y)}{[2(a_1^2+\mathbf{k}_\perp^2)(a_2^2+\mathbf{k}_\perp^2)]^{1/2}}\varphi(x, \mathbf{k}_\perp)$	$\frac{(a_1a_2-\mathbf{k}_\perp^2)}{[2(a_1^2+\mathbf{k}_\perp^2)(a_2^2+\mathbf{k}_\perp^2)]^{1/2}}\varphi(x, \mathbf{k}_\perp)$
$\lambda_1\lambda_2$	$\downarrow\uparrow$	$\downarrow\downarrow$
$\Psi_{\lambda_1\lambda_2}(x, \mathbf{k}_\perp, \lambda)$	$-\frac{(a_1a_2-\mathbf{k}_\perp^2)}{[2(a_1^2+\mathbf{k}_\perp^2)(a_2^2+\mathbf{k}_\perp^2)]^{1/2}}\varphi(x, \mathbf{k}_\perp)$	$-\frac{(a_1+a_2)(k_x+ik_y)}{[2(a_1^2+\mathbf{k}_\perp^2)(a_2^2+\mathbf{k}_\perp^2)]^{1/2}}\varphi(x, \mathbf{k}_\perp)$

our notations, we separate the spin-space wave function $\chi^K(x, \mathbf{k}_\perp, \lambda)$ out from the whole LC wave function, i.e., $\Psi^{(1-x)Q}(x, \mathbf{k}_\perp, \lambda) \to \chi^K(x, \mathbf{k}_\perp, \lambda)\varphi^{(1-x)Q}(x, \mathbf{k}_\perp)$, where $\Psi^{((1-x)Q)}(x, \mathbf{k}_\perp)$ is the light-cone wave function of the valence Fock state with a cut-off $|\mathbf{k}_\perp|$ of order $(1-x)Q$ and the spin space wave function $\chi^K(x, \mathbf{k}_\perp, \lambda)$ that comes from the spin space Wigner rotation can be found in Ref.[14] [1], which is given in TAB.I. One can combine the spin-space wave function $\chi^K(x, \mathbf{k}_\perp, \lambda)$ into the original T_H to form a new one, i.e.,

$$\begin{aligned} T_H \;=\; & (e_1\xi_1 + e_2\xi_2)T_H^{(\lambda_1+\lambda_2=0)}(\uparrow\downarrow\to\uparrow\downarrow) + (e_1\xi_1 + e_2\xi_2)T_H^{(\lambda_1+\lambda_2=0)}(\downarrow\uparrow\to\downarrow\uparrow) + \\ & (e_1\xi_1' + e_2\xi_2')T_H^{(\lambda_1+\lambda_2=1)}(\uparrow\uparrow\to\uparrow\uparrow) + (e_1\xi_1'^* + e_2\xi_2'^*)T_H^{(\lambda_1+\lambda_2=-1)}(\downarrow\downarrow\to\downarrow\downarrow)\;, \end{aligned} \tag{7}$$

where $\lambda_{1,2}$ are the helicities for the (initial or final) kaon's two constitute quarks respectively. It is found that there is no hard scattering amplitude with quark and antiquark helicities being changed due to the fact that the quark helicity is conserved at each quark-gluon (photon)-quark vertex in the limit of vanishing quark mass. e_i is the electric charge of the struck quark, ξ_i and ξ_i' are coefficients derived from $\chi^K(x, \mathbf{k}_\perp, \lambda)$,

$$\begin{aligned} \xi_1 &= \frac{(a_1a_2 - \mathbf{k}_\perp^2)(a_1'a_2' - \mathbf{l}_\perp^2)}{2[(a_1^2 + \mathbf{k}_\perp^2)(a_2^2 + \mathbf{k}_\perp^2)(a_1'^2 + \mathbf{l}_\perp^2)(a_2'^2 + \mathbf{l}_\perp^2)]^{1/2}}, \\ \xi_1' &= \frac{(a_1 + a_2)(a_1' + a_2')(\mathbf{k}_\perp \cdot \mathbf{l}_\perp + i\mathbf{k}_\perp \times \mathbf{l}_\perp)}{2[(a_1^2 + \mathbf{k}_\perp^2)(a_2^2 + \mathbf{k}_\perp^2)(a_1'^2 + \mathbf{l}_\perp^2)(a_2'^2 + \mathbf{l}_\perp^2)]^{1/2}}, \\ \xi_2 &= \frac{(b_1b_2 - \mathbf{k}_\perp^2)(b_1'b_2' - \mathbf{l}_\perp^2)}{2[(b_1^2 + \mathbf{k}_\perp^2)(b_2^2 + \mathbf{k}_\perp^2)(b_1'^2 + \mathbf{l}_\perp^2)(b_2'^2 + \mathbf{l}_\perp^2)]^{1/2}} \end{aligned} \tag{8}$$

and

$$\xi_2' = \frac{(b_1 + b_2)(b_1' + b_2')(\mathbf{k}_\perp \cdot \mathbf{l}_\perp + i\mathbf{k}_\perp \times \mathbf{l}_\perp)}{2[(b_1^2 + \mathbf{k}_\perp^2)(b_2^2 + \mathbf{k}_\perp^2)(b_1'^2 + \mathbf{l}_\perp^2)(b_2'^2 + \mathbf{l}_\perp^2)]^{1/2}}\;, \tag{9}$$

[1] Setting $m_1 = m_2 = m_q$, we return to the results of the case of pion.

where

$$a_1 = xM_a + m_1, a_2 = (1-x)M_a + m_2$$
$$a'_1 = yM'_a + m_1, a'_2 = (1-y)M'_a + m_2$$
$$b_1 = xM_b + m_2, b_2 = (1-x)M_b + m_1$$
$$b'_1 = yM'_b + m_2, b'_2 = (1-y)M'_b + m_1$$
$$M_a^2 = \frac{m_1^2 + \mathbf{k}_\perp^2}{x} + \frac{m_2^2 + \mathbf{k}_\perp^2}{1-x}, M_a'^2 = \frac{m_1^2 + \mathbf{l}_\perp^2}{y} + \frac{m_2^2 + \mathbf{l}_\perp^2}{1-y}$$
$$M_b^2 = \frac{m_2^2 + \mathbf{k}_\perp^2}{x} + \frac{m_1^2 + \mathbf{k}_\perp^2}{1-x}, M_b'^2 = \frac{m_2^2 + \mathbf{l}_\perp^2}{y} + \frac{m_1^2 + \mathbf{l}_\perp^2}{1-y}$$

Consequently, the above coefficients can be further simplified as

$$\begin{aligned}
\xi_1 &= \frac{[m_1(1-x)+m_2x][m_1(1-y)+m_2y]}{2\sqrt{k_\perp^2 + [m_1(1-x)+m_2x]^2}\sqrt{l_\perp^2 + [m_1(1-y)+m_2y]^2}}, \\
\xi'_1 &= \frac{(\mathbf{k}_\perp \cdot \mathbf{l}_\perp + i\mathbf{k}_\perp \times \mathbf{l}_\perp)}{2\sqrt{k_\perp^2 + [m_1(1-x)+m_2x]^2}\sqrt{l_\perp^2 + [m_1(1-y)+m_2y]^2}}, \\
\xi_2 &= \frac{[m_2(1-x)+m_1x][m_2(1-y)+m_1y]}{2\sqrt{k_\perp^2 + [m_2(1-x)+m_1x]^2}\sqrt{l_\perp^2 + [m_2(1-y)+m_1y]^2}}
\end{aligned} \tag{10}$$

and

$$\xi'_2 = \frac{(\mathbf{k}_\perp \cdot \mathbf{l}_\perp + i\mathbf{k}_\perp \times \mathbf{l}_\perp)}{2\sqrt{k_\perp^2 + [m_2(1-x)+m_1x]^2}\sqrt{l_\perp^2 + [m_2(1-y)+m_1y]^2}} \ . \tag{11}$$

Schematically, the total hard scattering amplitude can be written as

$$T_H = (e_1\xi_1 + e_2\xi_2)T_H^{(\lambda_1+\lambda_2=0)} + [e_1(\xi_1 + \xi_1'^*) + e_2(\xi_2 + \xi_2'^*)]T_H^{(\lambda_1+\lambda_2=\pm1)}$$

with

$$\begin{aligned}
T_H^{(\lambda_1+\lambda_2=0)} &= \frac{16\pi C_F \alpha_s(\mu_f^2)}{(1-x)(1-y)xy} \times (((x-1)\mathbf{q}_\perp{}^2 - 2\mathbf{k}_\perp \cdot \mathbf{q}_\perp)(2\mathbf{l}_\perp \cdot \mathbf{q}_\perp + (y-1)\mathbf{q}_\perp{}^2))^{-1} \\
&\quad ((x-1)(2\mathbf{l}_\perp \cdot \mathbf{q}_\perp + (y-1)\mathbf{q}_\perp{}^2) - 2(y-1)\mathbf{k}_\perp \cdot \mathbf{q}_\perp)^{-1} \times \\
&\quad \Big[2(y-1)y(1-y+x(2y-1))(\mathbf{k}_\perp \cdot \mathbf{q}_\perp)^2 + (x-1)x(2\mathbf{l}_\perp \cdot \mathbf{q}_\perp + (y-1)\mathbf{q}_\perp{}^2) \cdot \\
&\quad ((1-y+x(2y-1))(\mathbf{l}_\perp \cdot \mathbf{q}_\perp) + 2(x-1)(y-1)y\mathbf{q}_\perp{}^2) - \\
&\quad (x-1)(y-1)y(\mathbf{k}_\perp \cdot \mathbf{q}_\perp) \cdot (8x(\mathbf{l}_\perp \cdot \mathbf{q}_\perp) + (1-y+x(6y-5))\mathbf{q}_\perp{}^2)\Big],
\end{aligned} \tag{12}$$

and

$$T_H^{(\lambda_1+\lambda_2=\pm1)} = \frac{8\pi C_F \alpha_s(\mu_f^2)}{(1-x)(1-y)xy} \times (((x-1)\mathbf{q}_\perp{}^2 - 2\mathbf{k}_\perp \cdot \mathbf{q}_\perp)(2\mathbf{l}_\perp \cdot \mathbf{q}_\perp + (y-1)\mathbf{q}_\perp{}^2))^{-1}$$

$$((x-1)(2\mathbf{l}_\perp\cdot\mathbf{q}_\perp+(y-1)\mathbf{q}_\perp{}^2)-2(y-1)\mathbf{k}_\perp\cdot\mathbf{q}_\perp)^{-1}\Big[2(x-1)x(\mathbf{l}_\perp\cdot\mathbf{q}_\perp)^2+$$
$$(y-1)(2y(\mathbf{k}_\perp\cdot\mathbf{q}_\perp)^2+(x-1)(x(\mathbf{l}_\perp\cdot\mathbf{q}_\perp)-y(\mathbf{k}_\perp\cdot\mathbf{q}_\perp))\mathbf{q}_\perp{}^2)\Big], \tag{13}$$

where the scale $\mu_f^2 = Q^2$. It can be found that the leading contribution from the higher helicity components is of order $1/Q^4$, which is next-to-leading contribution compared to that of the ordinary helicity components.

With the help of Eq.(3), we can obtain the leading-twist hard part contribution to the kaon form factor. And after integrating over the azimuth angles for $\mathbf{k}_\perp$ and $\mathbf{l}_\perp$, we obtain the contribution from the usual helicity components ($\lambda_1+\lambda_2=0$),

$$\begin{aligned} F_K^{twist2,(\lambda_1+\lambda_2=0)}(Q^2) &= \int dxdyd\eta_1 d\eta_2 \frac{(e_1\xi_1)C_F\alpha_s(\mu_f^2)|\mathbf{k}_\perp||\mathbf{l}_\perp|}{32\pi^3 xy}\varphi(x,\mathbf{k}_\perp)\varphi^*(y,\mathbf{l}_\perp)\times \\ &\left[\frac{x(x+y-1-2xy)}{(1-x)\sqrt{1-\eta_1^2}}+\frac{y(x+y-1-2xy)}{(1-y)\sqrt{1-\eta_2^2}}+\frac{x+y-x^2-y^2}{(1-x)(1-y)\sqrt{1-\eta_1^2}\sqrt{1-\eta_2^2}}\right] \\ &+\Big\{e_1\leftrightarrow e_2,\ m_1\leftrightarrow m_2\Big\} \end{aligned} \tag{14}$$

and the contribution from the higher helicity components ($\lambda_1+\lambda_2=\pm1$),

$$\begin{aligned} F_K^{twist2,(\lambda_1+\lambda_2=\pm1)}(Q^2) &= -\int dxdyd\eta_1 d\eta_2 \frac{(e_1\xi_3)C_F\alpha_s(\mu_f^2)|\mathbf{k}_\perp||\mathbf{l}_\perp|}{64\pi^3 xy}\varphi(x,\mathbf{k}_\perp)\varphi^*(y,\mathbf{l}_\perp) \\ &\times\left[(x+y-2xy)\frac{\left(1-\sqrt{1-\eta_1^2}\right)\left(1-\sqrt{1-\eta_2^2}\right)}{(1-x)(1-y)\eta_1\eta_2\sqrt{1-\eta_1^2}\sqrt{1-\eta_2^2}}\right] \\ &+\Big\{e_1\leftrightarrow e_2,\ m_1\leftrightarrow m_2\Big\}, \end{aligned} \tag{15}$$

where without loss of generality, we have implicitly assumed that the radial kaon wave function $\varphi(x,\mathbf{k}_\perp)$ depending on $\mathbf{k}_\perp$ through $k_\perp^2$ only, i.e. $\varphi(x,\mathbf{k}_\perp)=\varphi(x,k_\perp^2)$. The terms in the big brace are obtained by transforming the terms out of the brace through the transformation $e_1\leftrightarrow e_2$ and $m_1\leftrightarrow m_2$. $\xi_3=\frac{|\mathbf{k}_\perp||\mathbf{l}_\perp|}{\sqrt{k_\perp^2+[m_1(1-x)+m_2x]^2}\sqrt{l_\perp^2+[m_1(1-y)+m_2y]^2}}$, $|\mathbf{k}_\perp|=Q(1-x)\eta_1/2$ and $|\mathbf{l}_\perp|=Q(1-y)\eta_2/2$, with $\eta_{1,2}$ in the range of $(0,\ 1)$. An overall minus sign in Eq.(15) implies that the higher helicity components shall always *suppress* the contribution from the usual helicity components.

B. Formulae for the twist-3 contributions to the kaon electromagnetic form factor

The end-point singularity becomes more serious for the higher twist structures, because the asymptotic behavior of the twist-3 DAs, especially $\phi_p^{as}(x) = 1$, so the calculations for these higher twist contributions have more uncertainty than that for the leading twist. As has been pointed out in Ref.[15], after including the parton transverse momenta, large double logarithmic corrections $\alpha_s \ln^2 k_\perp$ and $\alpha_s \ln^2 x$ appear in higher order radiative corrections and can be summed up to all orders. The relevant Sudakov form factors from both $k_\perp$ and the threshold resummation can cure the endpoint singularity and then the main contribution comes from the perturbative regions. For the present purpose, it is convenient to transform the kaon form factor into the compact parameter $\mathbf{b}$ space. In the large Q^2 region, by considering only the lowest valence quark state of the kaon and by doing the Fourier transformation of the wave function with the formula,

$$\Psi(x_i, \mathbf{k}_\perp; \mu_f) = \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{-i\mathbf{b}\cdot\mathbf{k}_\perp} \hat{\Psi}(x_i, \mathbf{b}; \mu_f),$$

we can transform the kaon electromagnetic form factor into the compact parameter $\mathbf{b}$ space,

$$\begin{aligned} F_K(Q^2) &= \int [dx_i d\mathbf{b}][dy_j d\mathbf{h}] \hat{\Psi}(x_i, \mathbf{b}; \mu_f) \hat{T}(x_i, \mathbf{b}; y_j, \mathbf{h}; \mu_f) \hat{\Psi}(y_j, \mathbf{h}; \mu_f) \times S_t(x_i) S_t(y_j) \times \\ & \exp(-S(x_i, y_j, Q, \mathbf{b}, \mathbf{h}; \mu_f)), \end{aligned} \tag{16}$$

where $\hat{\mu}_f = \ln(\mu_f/\Lambda_{QCD})$, $[dx_i d\mathbf{b}] = dx_1 dx_2 d^2\mathbf{b}\delta(1 - x_1 - x_2)/(16\pi^3)$ and the hard kernel

$$\hat{T}(x_i, \mathbf{b}; y_j, \mathbf{h}; \mu_f) = \int \frac{d^2\mathbf{k}_\perp}{(2\pi)^2} \frac{d^2\mathbf{l}_\perp}{(2\pi)^2} e^{-i\mathbf{b}\cdot\mathbf{k}_\perp - i\mathbf{h}\cdot\mathbf{l}_\perp} T(x_i, \mathbf{k}_{\perp i}; y_j, \mathbf{l}_{\perp j}; \mu_f).$$

The factor $\exp(-S(x_i, y_j, Q, \mathbf{b}, \mathbf{h}; \mu_f))$ contains the Sudakov logarithmic corrections and the renormalization group evolution effects of both the wave functions and the hard scattering amplitude,

$$S(x_1, y_1, Q, \mathbf{b}, \mathbf{h}; \mu_f) = \left[\left(\sum_{i=1}^{2} s(x_i, b, Q) + \sum_{j=1}^{2} s(y_j, h, Q)\right) - \frac{1}{\beta_1} \ln \frac{\hat{\mu}_f}{\hat{b}} - \frac{1}{\beta_1} \ln \frac{\hat{\mu}_f}{\hat{h}}\right], \tag{17}$$

where $\hat{b} \equiv \ln(1/b\Lambda_{QCD})$, $\hat{h} \equiv \ln(1/h\Lambda_{QCD})$ and $s(x, b, Q)$ is the Sudakov exponent factor, whose explicit form up to next-to-leading log approximation can be found in Ref.[16]. In Eq.(16), $S_t(x_i)$ and $S_t(y_i)$ come from the threshold resummation effects and the exact form of each involves one parameter integration [17]. In order to simplify the numerical calculations,

we take a simple parametrization proposed in Ref.[17],

$$S_t(x) = \frac{2^{1+2c}\Gamma(3/2+c)}{\sqrt{\pi}\Gamma(1+c)}[x(1-x)]^c \ , \tag{18}$$

where the parameter c is determined around 0.3.

With the help of the above equations, we obtain the formula for the twist-3 contributions to the kaon electromagnetic form factor,

$$\begin{aligned} F_K^{twist3}(Q^2) \ = \ & \frac{128\pi\mu_K^2}{3}\int_0^1 dxdy\int_0^\infty bdbhdh\alpha_s(\mu_f)\hat{T}(x,\mathbf{b};y,\mathbf{h};\mu_f)S_t(x_i)S_t(y_j) \times \\ & \left[y\hat{\Psi}_p(x,b;\mu_f)\hat{\Psi}_p^*(y,h;\mu_f) + (1+\bar{y})\hat{\Psi}_p(x,b;\mu_f)\frac{\hat{\Psi}_\sigma^{\prime *}(y,h;\mu_f)}{6} + \right. \\ & \left. \hat{\Psi}_p(x,b;\mu_f)\frac{\hat{\Psi}_\sigma^*(y,h;\mu_f)}{2}\right]\exp\left[-S(x_i,y_j,Q,\mathbf{b},\mathbf{h};\mu_f)\right] \ , \end{aligned} \tag{19}$$

where $\bar{x} = (1-x)$, $\bar{y} = (1-y)$ and $\hat{\psi}_\sigma^{\prime *}(y,h;\mu_f) = \partial\hat{\psi}_\sigma^*(y,h;\mu_f)/\partial y$. The hard scattering amplitude $\hat{T}(x,\mathbf{b};y,\mathbf{h};\mu_f)$ is given by

$$\begin{aligned} \hat{T}(x,\mathbf{b};y,\mathbf{h};\mu_f) \ = \ & K_0\left(\sqrt{\bar{x}\bar{y}}Qb\right)\Big(\theta(b-h)K_0\left(\sqrt{\bar{y}}Qb\right)I_0\left(\sqrt{\bar{y}}Qh\right) + \\ & \theta(h-b)K_0\left(\sqrt{\bar{y}}Qh\right)I_0\left(\sqrt{\bar{y}}Qb\right)\Big), \end{aligned} \tag{20}$$

where the higher power suppressed terms such as $({\mathbf{k}_\perp}^2/Q^2)$ has been neglected in the numerator, I_0 and K_0 are the modified Bessel functions of the first kind and the second kind respectively. To ensure that the pQCD approach is really applicable, one has to specify carefully the renormalization scale μ_f in the strong coupling constant. Here we take the scheme that is proposed in Refs.[18], i.e. its value is taken as the largest renormalization scale associated with the exchanged virtual gluon in the longitudinal and transverse degrees,

$$\mu_f = \max(\sqrt{\bar{x}\bar{y}}Q, 1/b, 1/h). \tag{21}$$

The full form of the kaon LC wave function have four helicity components (Table. I): namely,

$$\Psi = (\Psi_{\uparrow\uparrow}, \Psi_{\uparrow\downarrow}, \Psi_{\downarrow\uparrow}, \Psi_{\downarrow\downarrow}), \quad (\Psi = \Psi_p,\ \Psi_\sigma) \tag{22}$$

By including the higher helicity components, Eq.(19) can be improved as

$$\begin{aligned} F_K^{twist3}(Q^2) \ = \ & \frac{128\pi\mu_K^2}{3}\int_0^1 dxdy\int_0^\infty bdbhdh\alpha_s(\mu_f)\times\hat{T}(x,\mathbf{b};y,\mathbf{h};\mu_f)S_t(x_i)S_t(y_j) \times \\ & \left[y\sum_{\lambda_1\lambda_2}\mathcal{P}(\hat{\Psi}_p,\lambda_1,\lambda_2) + \frac{(1+\bar{y})}{6}\sum_{\lambda_1\lambda_2}\mathcal{P}(\hat{\Psi}'_\sigma,\lambda_1,\lambda_2) + \frac{1}{2}\sum_{\lambda_1\lambda_2}\mathcal{P}(\hat{\Psi}_\sigma,\lambda_1,\lambda_2)\right] \\ & \times\exp\left[-S(x_i,y_j,Q,\mathbf{b},\mathbf{h};\mu_f)\right], \end{aligned} \tag{23}$$

where $\hat{\Psi}'_\sigma = \partial\hat{\Psi}_\sigma/\partial x$ and

$$\sum_{\lambda_1\lambda_2} \mathcal{P}(\hat{\Psi}_p, \lambda_1, \lambda_2) = (\hat{\Psi}^*_{p\uparrow\downarrow}\hat{\Psi}_{p\uparrow\downarrow} + \hat{\Psi}^*_{p\downarrow\uparrow}\hat{\Psi}_{p\downarrow\uparrow}) - (\hat{\Psi}^*_{p\uparrow\uparrow}\hat{\Psi}_{p\uparrow\uparrow} + \hat{\Psi}^*_{p\downarrow\downarrow}\hat{\Psi}_{p\downarrow\downarrow}),$$

$$\sum_{\lambda_1\lambda_2} \mathcal{P}(\hat{\Psi}'_\sigma, \lambda_1, \lambda_2) = (\hat{\Psi}^*_{p\uparrow\downarrow}\hat{\Psi}'_{\sigma\uparrow\downarrow} + \hat{\Psi}^*_{p\downarrow\uparrow}\hat{\Psi}'_{\sigma\downarrow\uparrow}) - (\hat{\Psi}^*_{p\uparrow\uparrow}\hat{\Psi}'_{\sigma\uparrow\uparrow} + \hat{\Psi}^*_{p\downarrow\downarrow}\hat{\Psi}'_{\sigma\downarrow\downarrow}),$$

$$\sum_{\lambda_1\lambda_2} \mathcal{P}(\hat{\Psi}_\sigma, \lambda_1, \lambda_2) = (\hat{\Psi}^*_{p\uparrow\downarrow}\hat{\Psi}_{\sigma\uparrow\downarrow} + \hat{\Psi}^*_{p\downarrow\uparrow}\hat{\Psi}_{\sigma\downarrow\uparrow}) - (\hat{\Psi}^*_{p\uparrow\uparrow}\hat{\Psi}_{\sigma\uparrow\uparrow} + \hat{\Psi}^*_{p\downarrow\downarrow}\hat{\Psi}_{\sigma\downarrow\downarrow}).$$

For the hard scattering amplitude $\hat{T}(x,\mathbf{b};y,\mathbf{h};\mu_f)$, we have implicitly adopted the approximate relation, i.e. $\hat{T}(x,\mathbf{b};y,\mathbf{h};\mu_f)^{\uparrow\uparrow+\downarrow\downarrow} \approx -\hat{T}(x,\mathbf{b};y,\mathbf{h};\mu_f)^{\uparrow\downarrow+\downarrow\uparrow}$, since it can be found that

$$\hat{T}(x,\mathbf{b};y,\mathbf{h};\mu_f)^{\uparrow\uparrow+\downarrow\downarrow} = -\hat{T}(x,\mathbf{b};y,\mathbf{h};\mu_f)^{\uparrow\downarrow+\downarrow\uparrow} + \mathcal{O}(1/Q^2). \tag{24}$$

III. NUMERICAL RESULTS FOR THE KAON ELECTROMAGNETIC FORM FACTOR

Based on the formulae presented in the last section, we discuss sequentially the leading and the power suppressed contributions to the kaon electromagnetic form factor within the space-like region in the following. The differences between the pion and kaon electromagnetic form factors shall also be discussed in the due places. In the numerical calculations, we use $\Lambda^{(n_f=4)}_{\overline{MS}} = 250MeV$. As for the phenomenological parameter μ_K, which is a scale characterized by the chiral perturbation theory, we take its value to be $\mu_K \simeq 1.70$ GeV. And for definiteness, we take the conventional values for the constitute quark masses: $m_{u,d} = 0.30$GeV and $m_s = 0.45$GeV.

A. LC wave function of the kaon

In order to obtain the numerical results for the kaon electromagnetic form factor, we need to know its LC wave functions. One useful way is to use the approximate bound state solution of a hadron in terms of the quark model as the starting point for modeling the hadronic valence wave function. In combination of the spin-space wave function χ that comes from the Wigner rotation [19], the full form of the kaon LC wave function can be written as, $\Psi(x,\mathbf{k}_\perp,\lambda) = \varphi(x,\mathbf{k}_\perp)\chi$. The explicit form of the spin-space wave function χ

can be found in TAB.I. As for the radial part of the wave function, we adopt the model constructed in Refs.[20, 21], which is based on the BHL-prescription [12],

$$\begin{aligned}
\varphi_K(x,\mathbf{k}_\perp) &= [1+B_K C_1^{3/2}(2x-1)+C_K C_2^{3/2}(2x-1)]\frac{A_K}{x(1-x)} \\
&\quad \times \exp\left[-\beta_K^2\left(\frac{k_\perp^2+m_1^2}{x}+\frac{k_\perp^2+m_2^2}{1-x}\right)\right], \qquad (25)\\
\varphi_\sigma(x,\mathbf{k}_\perp) &= A_\sigma \exp\left[-\beta_K^2\left(\frac{k_\perp^2+m_1^2}{x}+\frac{k_\perp^2+m_2^2}{1-x}\right)\right], \qquad (26)\\
\varphi_p(x,\vec{k_\perp}) &= [1+B_p C_1^{1/2}(2x-1)+C_p C_2^{1/2}(2x-1)]\frac{A_p}{x(1-x)} \\
&\quad \times \exp\left[-\beta_K^2\left(\frac{k_\perp^2+m_1^2}{x}+\frac{k_\perp^2+m_2^2}{1-x}\right)\right], \qquad (27)
\end{aligned}$$

where $m_1=m_u$ and $m_2=m_s$ for $F_{K^+}(Q^2)$, $m_1=m_d$ and $m_2=m_s$ for $F_{K^0}(Q^2)$. $C_n^{3/2}(2x-1)$ and $C_n^{1/2}(2x-1)$ are Gegenbauer polynomials. A more complicated model that is also based on BHL-prescription is suggested in Ref.[5]. Numerically it can be found that the two model wave functions behave very likely under the same constraints. Additionally, it has argued in Ref.[22] that an extra factor $\sqrt{\partial k_z/\partial x}$ should be included into the LC wave function, otherwise, one can not obtain the right asymptotic behavior for the pion electromagnetic form factor at $Q^2\to\infty$. However, we have checked that without this factor, one can still obtain the right power behavior for the pion electromagnetic form factor as shown in Ref.[2], and such a factor will not bring any new features for the LC wave function if its parameters are determined properly. So we shall adopt the simpler form as suggested in Refs.[20, 21] to do our calculation.

The four parameters A_K, B_K, C_K and β_K of $\varphi_K(x,\mathbf{k}_\perp)$ can be determined by the first two Gegenbauer moments a_1^K and a_2^K of $\phi_K(x)$, the constraint $\langle\mathbf{k}_\perp^2\rangle_K^{1/2}\approx 0.350$GeV [23] and the normalization condition

$$\begin{aligned}
&\int_0^1 dx\int_{k_\perp^2<\mu_0^2}\frac{d^2\mathbf{k}_\perp}{16\pi^3}\frac{a_1a_2-\mathbf{k}_\perp^2}{[(a_1^2+\mathbf{k}_\perp^2)(a_2^2+\mathbf{k}_\perp^2)]^{1/2}}\varphi_K(x,\mathbf{k}_\perp)\\
&=\int_0^1 dx\int_{k_\perp^2<\mu_0^2}\frac{d^2\mathbf{k}_\perp}{16\pi^3}\frac{m_1(1-x)+m_2x}{\sqrt{k_\perp^2+[m_1(1-x)+m_2x]^2}}\varphi_K(x,\mathbf{k}_\perp)=\frac{f_K}{2\sqrt{6}}, \qquad (28)
\end{aligned}$$

where μ_0 stands for some hadronic scale that is of order $\mathcal{O}(1GeV)$. Here, the wave function is normalized to $f_K/2\sqrt{6}$ only for convenience, which is different from that of Refs.[20, 21] that is normalized to one [2], where the factor $f_K/2\sqrt{6}$ has been absorbed into the hard part

[2] It is noted that the unit of A_K in Ref.[20] should be corrected from GeV^{-1} to GeV^{-2}.

of the $B \to K$ transition form factor. And the average value of the transverse momentum square of kaon is defined as

$$\langle \mathbf{k}_\perp^2 \rangle_K = \frac{\int dx d^2\mathbf{k}_\perp |\mathbf{k}_\perp^2| |\Psi_K(x,\mathbf{k}_\perp)|^2}{\int dx d^2\mathbf{k}_\perp |\Psi_K(x,\mathbf{k}_\perp)|^2} = \frac{\int dx d^2\mathbf{k}_\perp |\mathbf{k}_\perp^2| |\varphi_K(x,\mathbf{k}_\perp)|^2}{\int dx d^2\mathbf{k}_\perp |\varphi_K(x,\mathbf{k}_\perp)|^2}.$$

The first Gegenbauer moment a_1^K has been studied by the light-front quark model [24], the LCSR approach [25, 26, 27, 28, 29] and the lattice calculation [30, 31, 32] and etc. The higher Gegenbauer moments, such as a_2^K, are still determined with large uncertainty [25, 26, 27, 33, 34, 35]. In the following calculation, if not specially stated, we take $a_1^K(1\text{GeV}) = 0.05$ [26] and $a_2^K(1GeV) = 0.115$ [35] to be their default values, and shall discuss the uncertainties caused by these two factors in due places. By taking these default values, we obtain

$$A_K = 12.55 GeV^{-1},\ B_K = 0.0605,\ C_K = 0.0348,\ \beta_K = 0.8706 GeV^{-1}\ . \tag{29}$$

The parameter A_σ of $\varphi_\sigma(x,\mathbf{k}_\perp)$ can be determined by its normalization condition similar to Eq.(28), i.e. $A_\sigma = 65.04 GeV^{-1}$. And the coefficients A_p, B_p and C_p of $\varphi_p(x,\mathbf{k}_\perp)$ can be determined by the DA moments of $\varphi_p(x,\mathbf{k}_\perp)$. To discuss the uncertainty caused by Ψ_p, we take two groups of DA moments that have been obtained in Refs.[25, 36] to determine the coefficients A_p, B_p and C_p, where the moments in Ref.[25] are derived by using the QCD light-cone sum rules and the moments in Ref.[36] are derived based on the effective chiral action from the instanton:

$$\text{Group 1}\quad [25]:\quad \langle x^0\rangle_p^K = 1,\ \ \langle x^1\rangle_p^K = 0.06124,\ \ \langle x^2\rangle_p^K = 0.36757, \tag{30}$$

$$\text{Group 2}\quad [36]:\quad \langle x^0\rangle_p^K = 1,\ \ \langle x^1\rangle_p^K = 0.00678,\ \ \langle x^2\rangle_p^K = 0.35162. \tag{31}$$

Here the moments are defined as $\langle x^i\rangle_p^K = \int_0^1 dx(2x-1)^i \phi_p(1-x,\mu_0)$ with $i = (0,1,2)$. It should be noted that the moments defined in Ref.[25, 36] are for $\phi_p(1-x,\mu_0)$ other than $\phi_p(x,\mu_0)$, since in these references x stands for the momentum fraction of s-quark in the kaon ($\bar{K}$), while in the present paper x stands for the momentum fraction of the light quark q in the kaon (K). Taking the above two groups of DA moments, the parameters of $\varphi_p(x,\vec{k_\perp})$ can be determined as,

$$\text{Group 1}:\qquad A_p^1 = 12.12\text{GeV}^{-1},\quad B_p^1 = 0.3062,\quad C_p^1 = 1.604, \tag{32}$$

$$\text{Group 2}:\qquad A_p^2 = 12.04\text{GeV}^{-1},\quad B_p^2 = 0.4711,\quad C_p^2 = 1.506. \tag{33}$$

It is found that both distribution amplitudes are double humped curves and are highly suppressed in the endpoint region. Such feature is necessary to suppress the endpoint singularity

coming from the hard-scattering kernel and then to derive a more reasonable results for the twist-3 contributions to the kaon electromagnetic form factor.

B. Valence Fock state contribution in the low energy region

At the present, the experimental data on the kaon electromagnetic form factor are concentrated in the low energy region $Q^2 < 1GeV^2$, c.f. Ref.[11]. The soft part contribution can be written as

$$F_K^s(Q^2) = \sum_{\lambda,\lambda'} \sum_j e_j \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \Psi_K^*(x, \mathbf{k}_\perp, \lambda) \Psi_K(x, \mathbf{k}'_\perp, \lambda') + \cdots, \tag{34}$$

where λ, λ' are the helicities of the wave function respectively, and the first term is the lowest order contribution from the minimal Fock state (valence Fock state) and the ellipses represent those from higher Fock states, which are down by powers of $1/Q^2$ and by powers of α_s in the large Q^2 region. In general, the kaon electromagnetic form factor should sum over all of higher Fock state contributions in the low energy region. If only taking the leading-twist LC wave function of the valence Fock state, we can examine the contribution from the valence Fock state in the low energy region, i.e.

$$F_K^{s(V)}(Q^2) = e_1 \left[\int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \kappa \varphi_K(x, \mathbf{k}_\perp) \varphi_K^*(x, \mathbf{k'}_\perp) \right] + e_2 \left[m_1 \leftrightarrow m_2 \right], \tag{35}$$

where $e_1 = 2/3$, $e_2 = 1/3$, $m_1 = m_u$ and $m_2 = m_s$ for $F_{K^+}^{s(V)}$ and $e_1 = -1/3$, $e_2 = 1/3$, $m_1 = m_d$ and $m_2 = m_s$ for $F_{K^0}^{s(V)}$ respectively. The terms in the second bracket is obtained by transforming the terms in the first bracket through $m_1 \leftrightarrow m_2$, the coefficient κ that is from the spin-space Winger rotation can be written as

$$\begin{aligned}
\kappa &= \frac{(a_1 a_2 - \mathbf{k}_\perp^2)(a'_1 a'_2 - \mathbf{k'}_\perp^2) + (a_1 + a_2)(a'_1 + a'_2)\mathbf{k}_\perp \cdot \mathbf{k'}_\perp}{[(a_1^2 + \mathbf{k}_\perp^2)(a_2^2 + \mathbf{k}_\perp^2)(a_1'^2 + \mathbf{k'}_\perp^2)(a_2'^2 + \mathbf{k'}_\perp^2)]^{1/2}} \\
&= \frac{[m_1(1-x) + m_2 x]^2 + \mathbf{k}_\perp \cdot \mathbf{k'}_\perp}{\sqrt{k_\perp^2 + [m_1(1-x) + m_2 x]^2}\sqrt{k_\perp'^2 + [m_1(1-x) + m_2 x]^2}},
\end{aligned} \tag{36}$$

where $\mathbf{k'}_\perp = \mathbf{k}_\perp + (1-x)\mathbf{q}_\perp$ for the final state LC wave function when taking the Drell-Yan-West assignment [6]. Since $m_1 \neq m_2$, we have $F_{K^0}^{s(V)}(Q^2) \neq 0$, which is different from the pionic case, i.e. $F_{\pi^0}^{s(V)}(Q^2) \equiv 0$ because π^0 has equal quark masses.

We proceed to integrate the transverse momentum $\mathbf{k}_\perp$ in Eq.(35) with the help of the Schwinger $\alpha-$representation method,

$$\frac{1}{A^\kappa} = \frac{1}{\Gamma(\kappa)} \int_0^\infty \alpha^{\kappa-1} e^{-\alpha A} d\alpha \ . \tag{37}$$

Doing the integration over $\mathbf{k}_\perp$, we obtain

$$\begin{aligned}
&\int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \kappa \varphi_K(x,\mathbf{k}_\perp)\varphi_K^*(x,\mathbf{k}'_\perp) \\
=& \int_0^1 dx \int_0^\infty d\lambda \frac{A_K^2(x)}{128\pi^2(1+\lambda)^3} \times \left[I_0\left(\frac{Q^2(x-1)\beta_K^2\lambda^2}{4x(1+\lambda)}\right)\left(\frac{4(1-x)x(1+\lambda)}{\beta_K^2}\right.\right. \\
&\left.\left. -Q^2(1-x)^2(2+\lambda(4+\lambda)) + 8m_b^2(1+\lambda)^2\right) - I_1\left(\frac{Q^2(x-1)\beta_K^2\lambda^2}{4x(1+\lambda)}\right) Q^2(1-x)^2\lambda^2 \right] \\
&\times \exp\left[-\frac{\beta_K^2[8m_a^2(1+\lambda)+8m_b^2\lambda(\lambda+1)+Q^2(1-x)^2(2+\lambda(4+\lambda))]}{4(1-x)x(1+\lambda)} \right],
\end{aligned} \tag{38}$$

where the short notations $A_K(x) \equiv \frac{A_K}{x(1-x)}[1+B_K C_1^{3/2}(2x-1)+C_K C_2^{3/2}(2x-1)]$, $m_a^2 = m_1^2(1-x)+m_2^2 x$, $m_b^2 = [m_1(1-x)+m_2 x]^2$, and I_n $(n=0,1)$ is the modified Bessel function of the first kind. Substituting Eq.(38) into Eq.(35) and doing the expansion in the small Q^2 limit, we obtain

$$\begin{aligned}
&F_K^{s(V)}(Q^2)|_{Q^2=0} \\
=& \ e_1 \left\{ \int_0^1 dx \int_0^\infty d\lambda \frac{A_K^2(x)}{16\pi^2(1+\lambda)^2} \exp\left[-\frac{2\beta_K^2(m_a^2+m_b^2\lambda)}{(1-x)x} \right] \left[m_b^2(1+\lambda) + \frac{x(1-x)}{2\beta_K^2} \right] \right\} \\
&+e_2 \Big\{ m_1 \leftrightarrow m_2 \Big\},
\end{aligned} \tag{39}$$

where the term $[m_b^2(1+\lambda)]$ in the second square bracket comes from the ordinal helicity components, while the remaining terms are from the higher helicity components. As for the mean square radius $\langle r_K^2 \rangle_V$, we obtain

$$\begin{aligned}
\langle r_K^2 \rangle_V \ \approx& \ -6 \left. \frac{\partial F_K^{s(V)}(Q^2)}{\partial Q^2} \right|_{Q^2=0} \\
=& \ e_1 \left\{ \int_0^1 dx \int_0^\infty d\lambda \frac{3A_K^2(x)\beta_K^2}{32\pi^2 x(1+\lambda)^3} \exp\left[-\frac{2\beta_K^2(m_a^2+\lambda m_b^2)}{(1-x)x} \right] (1-x)(2+4\lambda+\lambda^2) \right. \\
&\left. \times \left[\frac{(1-x)x}{\beta_K^2} + m_b^2(1+\lambda) \right] \right\} + e_2 \Big\{ m_1 \leftrightarrow m_2 \Big\}.
\end{aligned} \tag{40}$$

The result for the soft contribution to the kaon electromagnetic form factor is shown in Fig.(2), where the solid line is for $a_1^K(1GeV) = 0.05$ and $a_2^K(1GeV) = 0.115$ and the shaded band is obtained with $a_1^K(1GeV) \in [0.03, 0.07]$ and $a_2^K(1GeV) \in [0.05, 0.10]$. The valence quark contribution is slightly below the experimental data, which means that there are still some space for the higher Fock state contributions. With the help of Eq.(38), we can estimate the probability of finding the valance states in the charged/neutral kaon, e.g. we obtain ($P_{u\bar{s}} = 0.901 < 1.0$), which shows that higher Fock states and higher twist terms

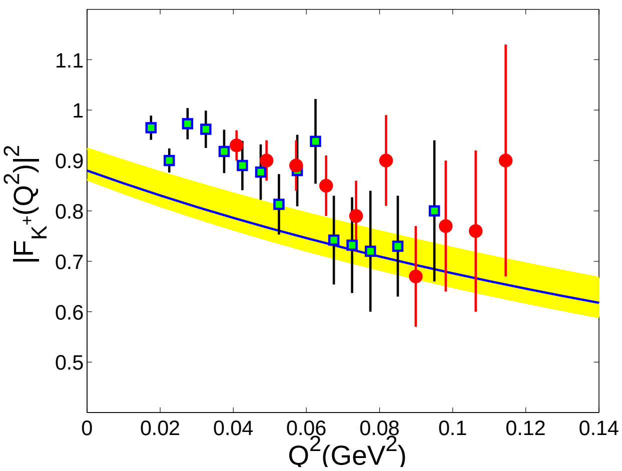

FIG. 2: The valence Fock state contribution to the kaon electromagnetic form factor $|F_{K^+}(Q^2)|^2$ in the low energy region, where the experimental data is taken from Ref.[37]. The shaded band is obtained with $a_1^K(1GeV) \in [0.03, 0.07]$ and $a_2^K(1GeV) \in [0.05, 0.10]$, and the solid line is for $a_1^K(1GeV) = 0.05$ and $a_2^K(1GeV) = 0.115$.

should also be considered to give a full understanding of the form factor at the energy region $Q^2 \to 0$. Such probability can be further divided into two parts: ($P_{u\bar{s}}^{(\lambda_1+\lambda_2=0)} = 0.562$) for the usual helicity components and ($P_{u\bar{s}}^{(\lambda_1+\lambda_2=\pm1)} = 0.339$) for the higher helicity states. It shows that the higher helicity components have the same importance as that of the usual helicity components in the soft energy region. It is noted that the higher helicity components' contribution to the kaon electromagnetic form factor has also been studied with the LC framework in Ref.[5], where the probability of the leading Fock state is just normalized to one and the experimental data on the mean-square radius of charged/neutral kaon are used to determine the wave function parameters. As argued above, this simple treatment maybe not right, since then the contribution from the valence state can be enhanced and become important inadequately [3].

As for the charged and neutral mean square radii $\langle r^2_{K^+}\rangle_V$ and $\langle r^2_{K^0}\rangle_V$, we obtain $\langle r^2_{K^\pm}\rangle_V^{1/2} = 0.570$ fm and $\langle r^2_{K^0}\rangle_V = -0.0736$ fm^2, which is consistent with the Ref.[38], while experimentally we have $\langle r^2_{K^\pm}\rangle^{1/2} = 0.560 \pm 0.031$ fm and $\langle r^2_{K^0}\rangle = -0.077 \pm 0.010$ fm^2 [11]. Further more, we give a simple estimation of the uncertainties caused by the two Gegen-

[3] As has been pointed out in Ref.[2], the condition for the pionic case is more serious, where the probabilities for the value quark state is only about 74%.

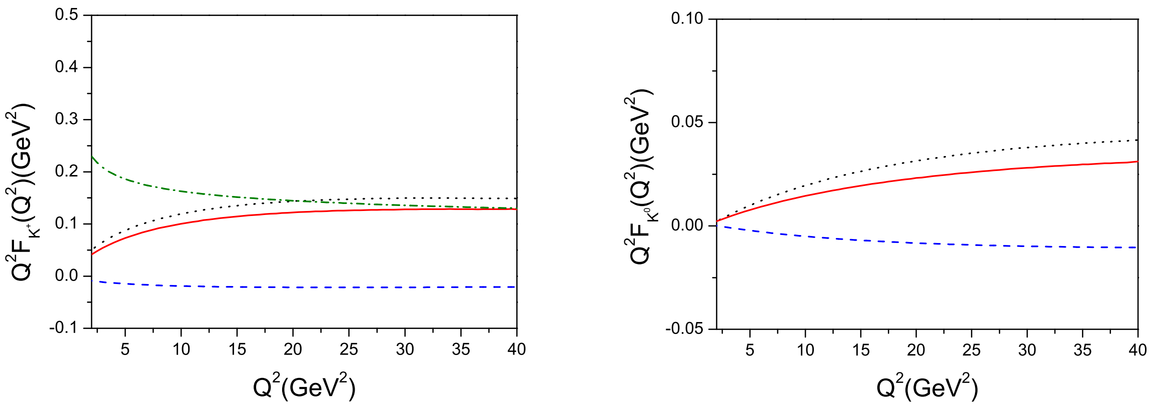

FIG. 3: Leading twist contribution to the kaon electromagnetic form factor in the intermediate and large energy region, where the left is for $Q^2 F_{K^+}(Q^2)$ and the right is for $Q^2 F_{K^0}(Q^2)$. The dotted line stands for the contribution from the usual helicity ($\lambda_1 + \lambda_2 = 0$) components, the dashed line stands for the contribution from the higher helicity ($\lambda_1 + \lambda_2 = \pm 1$) components and the solid line is the total hard contribution, which is the combined result for all the helicity components. The dash-dot line stands for the usual asymptotic result of charged kaon.

bauer moments of the kaon twist-2 wave function, e.g. by taking $a_1^K(1GeV) = 0.05 \pm 0.02$ and $a_2^K(1GeV) = 0.10 \pm 0.05$, we obtain the probability of finding the valence Fock state in the kaon $P_{u\bar{s}} = 0.901^{+0.026}_{-0.010}$, and the uncertainties of radii $\langle r^2_{K^\pm} \rangle_V^{1/2} = 0.570^{+0.021}_{-0.028}$ fm and $\langle r^2_{K^0} \rangle_V = -0.0736^{+0.018}_{-0.014}$ fm^2. It should be noted that by taking different values for a_1^K and a_2^K, all the undermined parameters of the wave function should be varied accordingly.

C. leading twist contribution to the kaon electromagnetic form factor

With the help of the LC wave function Eq.(25), we show the leading twist contribution to the kaon electromagnetic form factor in the intermediate and large energy region in Fig.(3), where the contribution from the usual helicity component or from the higher helicity components are considered. It is shown that the higher helicity components always suppress the usual helicity components' contributions to the kaon electromagnetic form factor. The usual asymptotic result of charged kaon, i.e. $Q^2 F_{K^+}(Q^2)|_{asy} = 8\pi f_K^2 \alpha_s(Q^2)$, is also presented in Fig.(3) for reference. It can be found that the leading contribution of the hard-scattering

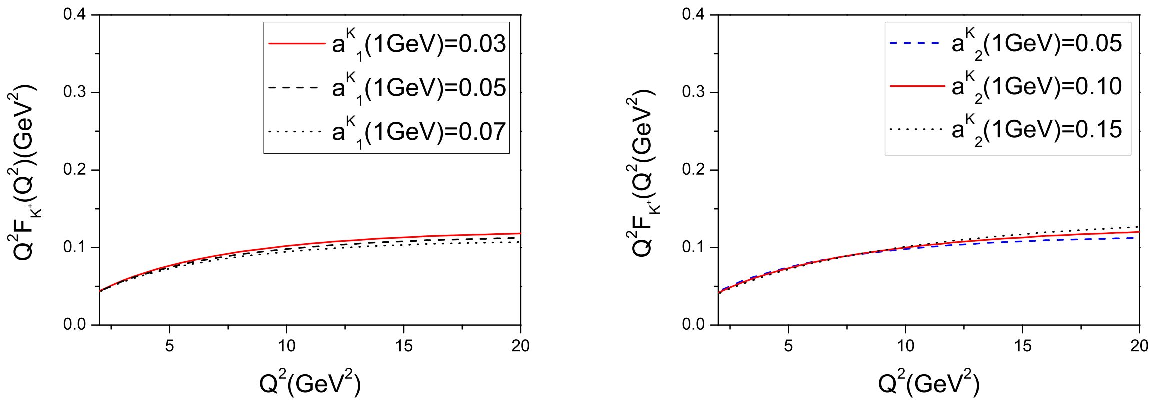

FIG. 4: Uncertainties of the leading twist contribution to $Q^2 F_{K^+}(Q^2)$ caused by a_1^K and a_2^K, where the left diagram is for fixed $a_2^K(1GeV) = 0.05$ with $a_1^K(1GeV) = 0.03$, 0.05 and 0.07, and the right diagram is for fixed $a_1^K(1GeV) = 0.05$ with $a_2^K(1GeV) = 0.05$, 0.10 and 0.15 respectively.

amplitude from the higher helicity components is of order $1/Q^4$, which is next-to-leading contribution compared to the contribution coming from the ordinary helicity component, but it shall give sizable contributions to the kaon electromagnetic form factor in the intermediate energy region. The net contribution shows the right power behavior $Q^2 F_{K^+}(Q^2)|_{Q^2\to\infty} \to$ const. In the present work, we have considered the k_T dependence both in the wave function and in the hard scattering amplitude consistently within the LC pQCD approach, then our results present a right power behavior for the higher helicity components' contributions. Secondly, in contrary to the pionic case, we obtain $F_{K^0}(Q^2) = -F_{\bar{K}^0}(Q^2) \neq 0$ at $Q^2 \neq 0$, which are rightly caused by the $SU_f(3)$-breaking effect and are strongly dependent on the constitute quark masses.

We take the charged kaon electromagnetic form factor as a concrete example to show the uncertain caused by $a_1^K(1GeV)$ and $a_2^K(1GeV)$, which are varied within the region of $[0.03, 0.07]$ and $[0.05, 0.15]$ respectively. We draw the charged kaon electromagnetic form factor in Fig.(4), where the left diagram is for fixed $a_2^K(1GeV) = 0.05$ with $a_1^K(1GeV) = 0.03$, 0.05 and 0.07, and the right diagram is for fixed $a_1^K(1GeV) = 0.05$ with $a_2^K(1GeV) = 0.05$, 0.10 and 0.15 respectively. $Q^2 F_{K^+}(Q^2)$ decreases with the increment of a_1^K. From Fig(4), it can be found that the uncertainty of the form factor caused by $a_1^K(1\mathrm{GeV}) = 0.05 \pm 0.02$ is small, e.g. it is about $\pm 5\%$ for $q^2 \in [2, 20] GeV^2$. And the uncertainty of the form factor

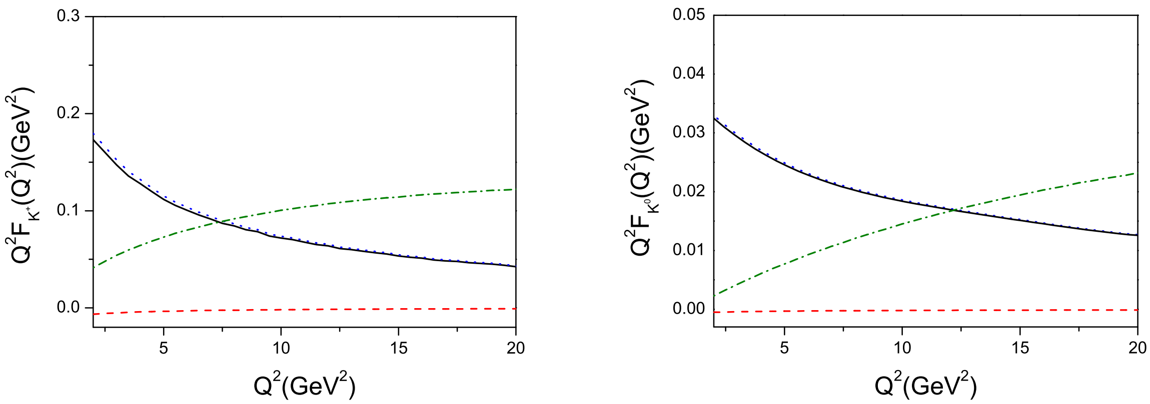

FIG. 5: Twist-3 contribution to the kaon electromagnetic form factor $Q^2F_{K^+}(Q^2)$ and $Q^2F_{K^0}(Q^2)$. The dotted line stands for the contribution from the usual helicity ($\lambda_1+\lambda_2=0$) components, the dashed line stands for the contribution from the higher helicity ($\lambda_1+\lambda_2=\pm1$) components and the solid line is the total hard contribution, which is the combined result for all the helicity components. As a comparison, the twist-2 contribution is shown in dash-dot line.

caused by $a_2^K(1\mathrm{GeV})$ varying within a bigger region $[0.05, 0.15]$ is also small, i.e. which is about $4\%-9\%$ for $q^2\in[2,20]GeV^2$. $Q^2F_{K^+}(Q^2)$ decreases with the increment of a_2^K in the lower energy region $q^2\lesssim 6GeV^2$ and increases with the increment of a_2^K in the higher energy region $q^2\gtrsim 6GeV^2$.

D. twist-3 contribution to the kaon electromagnetic form factor

We show the twist-3 contribution to kaon electromagnetic form factors $Q^2F_{K^+}(Q^2)$ and $Q^2F_{K^0}(Q^2)$ in Fig.(5), which are obtained with the full form of the LC wave functions $\Psi_p^f(x,\mathbf{k}_\perp)$ and $\Psi_\sigma^f(x,\mathbf{k}_\perp)$ and with the Group 1 parameters for $\Psi_p(x,\mathbf{k}_\perp)$. It is found that at the twist-3 level, the higher helicity components' contributions to the form factor are negative and small in comparison to that of the usual helicity components. The twist-2 contribution is also presented in Fig.(5) for comparison. At the twist-3 level, both the charged and the neutral kaon electromagnetic form factors decrease with the increment Q^2, and the charged form factor becomes smaller than the twist-2 contribution at around $Q^2=7GeV^2$, which is changed to be $Q^2\simeq 12GeV^2$ for the neutral case. This implies that the twist-3 contributions are sizable in the intermediate energy region and are rightly power suppressed to the twist-2

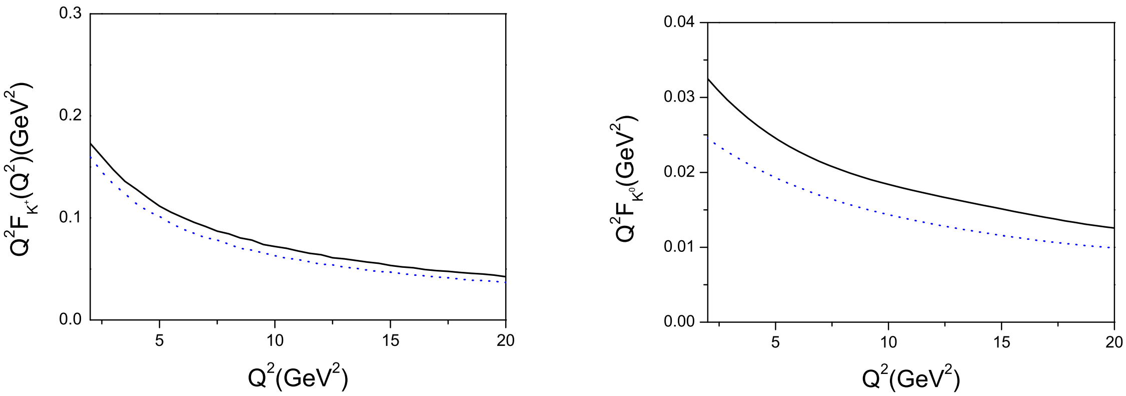

FIG. 6: Uncertainty caused by two types of twist-3 wave function Ψ_p to the electromagnetic form factors $Q^2F_{K^+}(Q^2)$ (Left diagram) and $Q^2F_{K^0}(Q^2)$ (Right diagram). The solid line and the dotted line are for Group 1 and Group 2 parameters respectively.

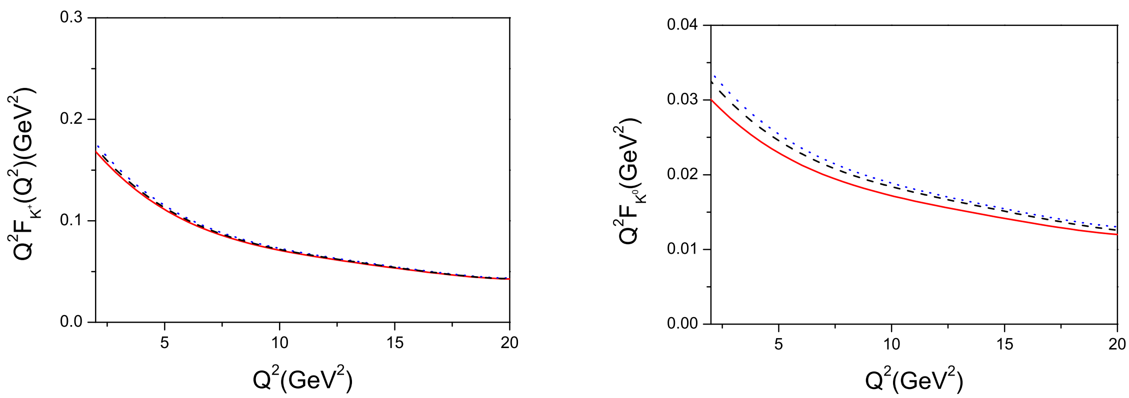

FIG. 7: Uncertainty caused by the parameter β_K of Ψ_p with Group 1 parameters to the electromagnetic form factors $Q^2F_{K^+}(Q^2)$ (Left diagram) and $Q^2F_{K^0}(Q^2)$ (Right diagram). The solid line, the dashed line and the dotted line are for $\beta_K = 0.85GeV^{-1}$, $0.87GeV^{-1}$ and $0.89GeV^{-1}$ respectively.

contributions in the large energy region, which is similar to the pionic case as shown in Ref.[1].

The main uncertainty sources for the twist-3 contribution come from the wave function $\Psi_p(x, \mathbf{k}_\perp)$ and the parameter β_K. We show the contributions to the charged kaon electromagnetic form factor from the two groups of parameters for $\Psi_p(x, \mathbf{k}_\perp)$ in Fig.(6), c.f. Eqs.(32,

33). It is found that the uncertainty within the allowable energy region caused by these two groups of parameters are about $10-20\%$ and $20-30\%$ for the charged case and the neutral case respectively. Secondly, we show the uncertainty caused by the parameter β_K in Fig.(7), where the Group 1 moments [25] are used to determine the parameters of $\Psi(x,\mathbf{k}_\perp)$ and three typical values $\beta_K = 0.85GeV^{-1}$, $0.87GeV^{-1}$ and $0.89GeV^{-1}$ are adopted [4]. The twist-3 contribution increases with the increment of β_K, and the uncertainty is less than 5% for the charged form factor, while for the neutral form factor the uncertainty changes to be $\sim 10\%$.

As for the higher order corrections, we present a naive estimation of the next-to-leading order (NLO) twist-2 contribution to the charged kaon electromagnetic form factor with the help of the asymptotic DA, i.e. with the renormalization scale and the factorization scale taken to be $\mu_R^2 = \mu_f^2 = Q^2$, it can roughly be expressed as [39], $Q^2 F_{K^+}^{NLO} \approx (0.903GeV^2)\frac{f_K^2}{f_\pi^2}\alpha_s^2(Q^2)$. Numerically the NLO correction will give about $\sim 20-30\%$ extra contribution to the charged kaon electromagnetic form factor.

IV. SUMMARY AND CONCLUSION

The k_T factorization formalism provides a convenient framework and has been widely applied to various processes. In this paper we present a systematical study on the kaon electromagnetic form factors $F_{K^\pm,K^0,\bar{K}^0}(Q^2)$ within the k_T factorization formalism. In order to get a deeper understanding of the hard contributions at the energy region where pQCD is applicable, we have examined the transverse momentum effects, the contributions from the different helicity components and different twist structures of the kaon LC wave function. Our results show that the right power behavior of the hard contribution from the higher helicity components and from the higher twist structures can be obtained by keeping the k_T dependence in the hard scattering amplitude. The full estimation of the power suppressed contributions to the kaon electromagnetic form factors $Q^2F_{K^+}(Q^2)$ and $Q^2F_{K^0}(Q^2)$ is shown in Fig.(8). The k_T dependence in LC wave function affects the hard and soft contributions substantially and the power-suppressed terms (twist-3 and higher helicity components) make an important contribution below $Q^2 \sim several\ GeV^2$ although

[4] When varying $a_1^K(1GeV)$ and $a_2^K(1GeV)$ within the region of $[0.03, 0.07]$ and $[0.05, 0.15]$ respectively, the value of β_K shall vary within the region of $[0.856, 0.896]GeV^{-1}$.

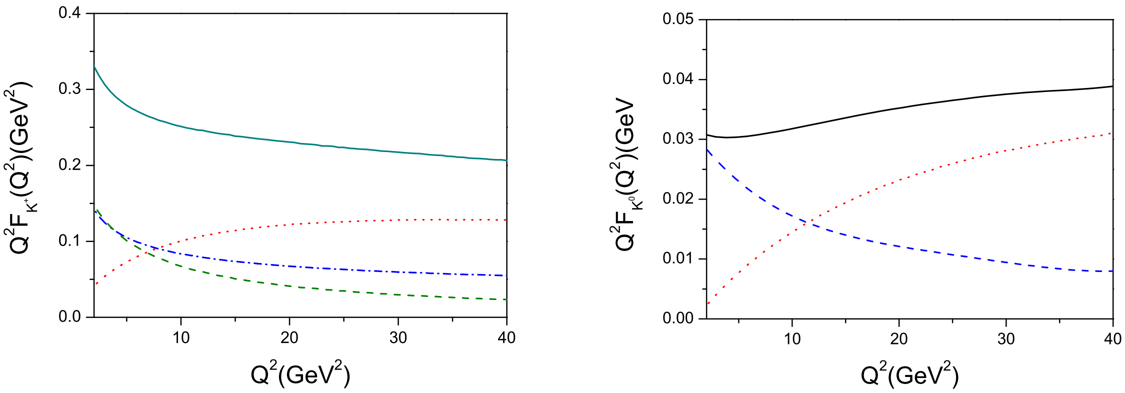

FIG. 8: The combined hard contribution for the kaon electromagnetic form factors $Q^2F_{K^+}(Q^2)$ (Left diagram) and $Q^2F_{K^0}(Q^2)$ (Right diagram). The solid line stands for the combined hard contribution, the dotted line, the dashed line and the dash-dot line are for LO twist-2, twist-3 and NLO twist-2 contributions respectively.

they drop fast as Q^2 increasing.

The parameters of the proposed model wave function can be fixed by the first two moments of its distribution amplitude and the normalization condition. In this paper we have taken the first two moments $a_1^K(1GeV) = 0.05 \pm 0.02$ and $a_2^K(1GeV) = 0.10 \pm 0.05$. It is found that the uncertainty of the kaon electromagnetic form factor, which is caused by varying values within the above range, is rather small. It is also found that the power-suppressed twist-3 contribution makes an important contribution at $Q^2 \sim several\ GeV^2$ and drops fast as Q^2 increasing. A naive estimation gives the NLO correction about $\sim 20 - 30\%$ extra contribution to the charged kaon form factor.

The relativistic effect due to the Wigner rotation have also been applied to calculation the kaon electromagnetic form factor. Consequently there are higher-helicity $(\lambda_1 + \lambda_2 = \pm 1)$ components in the spin space wave function besides the usual-helicity $(\lambda_1 + \lambda_2 = 0)$ components. It is shown that the higher helicity components have the same importance as that of the usual helicity components for the soft energy region, e.g. the probability of finding the valance states in the charged kaon includes two parts: $(P_{u\bar{s}}^{(\lambda_1+\lambda_2=0)} = 0.562)$ for the usual helicity components and $(P_{u\bar{s}}^{(\lambda_1+\lambda_2=\pm 1)} = 0.339)$ for the higher helicity states for $a_1^K(1GeV) = 0.05$ and $a_2^K(1GeV) = 0.115$. By taking $a_1^K(1GeV) = 0.05 \pm 0.02$ and $a_2^K(1GeV) = 0.10 \pm 0.05$, we obtain the uncertainty of the probabilities $P_{u\bar{s}} = 0.901^{+0.026}_{-0.010}$,

It is found that the hard-scattering amplitude for the higher-helicity components is of order $1/Q^4$ which is the next to leading contribution compared with the contribution coming from the ordinary helicity component, but it can give sizable contributions to the kaon electromagnetic form factor, especially for the twist-2 case as shown in Fig.(3).

Acknowledgements

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